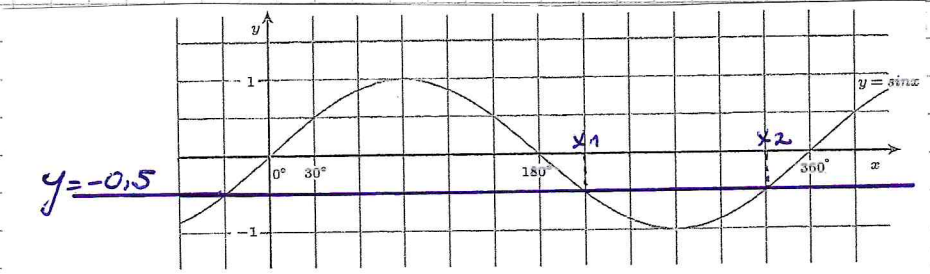
# Teema 7 iseseisvad

**1** a) lineaarfunktsiooni $y = -0{,}5$ graafik (on parallelne x-teljega ja läbib y-telge kohas $-0{,}5$)

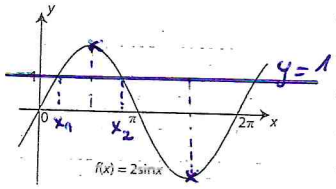



b) Võrrandi $\sin x = -0{,}5$ lahendid on:

$x_1 = 210°$ ; $x_2 = 330°$

**2** a) võrrandi $2\sin x = 1$ lahendid:

* joonestan $y = 1$ graafiku
* lahendid on: $x_1 = \frac{\pi}{6}$ ; $x_2 = \frac{5\pi}{6}$



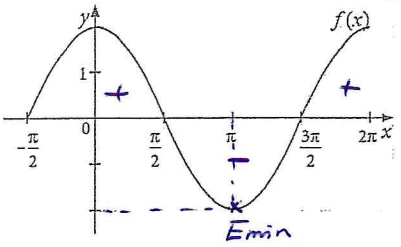

b) Kahaneminvahemik $X\downarrow = \left(\frac{\pi}{2}; \frac{3\pi}{2}\right)$

**3** a) * Negatiivsus piirkond

$X^- = \left(\frac{\pi}{2}; \frac{3\pi}{2}\right)$

* Miinimumpunkti koordinaadid

$E_{min}\left(\pi; -2\right)$




b) Kas punkt $A\left(\frac{\pi}{3}; -1\right)$ asub graafikul?

Kui $x = \frac{\pi}{3}$, siis $y = 2\cos\frac{\pi}{3} = 2\cos 60° = 1 \neq -1$

$\Rightarrow$ punkt A ei asu graafikul

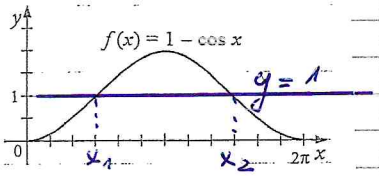
**4** a) $\sin\frac{\pi}{2} - \tan x \cdot \frac{\cos^2 x}{\sin x} = \sin 90° - \frac{\sin x}{\cos x} \cdot \frac{\cos^2 x}{\sin x} =$

$= 1 - \frac{\sin x \cdot \cos x}{\cos x \cdot \sin x} = \underline{1 - \cos x}$

b) Võrrandi $1 - \cos x = 1$ lahendid

* joonestan $y = 1$ graafiku

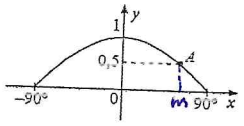
$x_1 = \frac{\pi}{2}$ ; $x_2 = \frac{3\pi}{2}$



(5) a) $\frac{1-\sin^2 x}{\sin(90°-x)} = \frac{\sin^2 x + \cos^2 x - \sin^2 x}{\cos x} = \frac{\cos x}{\cos x} = 1$ — $\cos x$ (2)

b) m väärtus.

$A(m; 0,5)$




Graafik $y = \cos m$

$\cos m = 0,5$

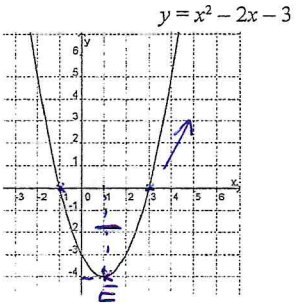
$m = 60°$, sest $\cos 60° = 0,5$

(6) a) muutumispiirkond

$Y = [-4; \infty)$

b) Negatiivsuspiirkond

$X^- = (-1; 3)$




c) Ekstreemumpunkti koordinaadid

$E_{min}(1; -4)$

d) Kasvamisvahemik

$X\uparrow = (1; \infty)$

(7) Funktsioon $f(x) = x^3 + 8x^2 - 12x + 7$

a) Tuletis $y' = 3x^2 + 16x - 12$

b) *ekstreemumkohad $x_e$: tingimus $y' = 0$

$3x^2 + 16x - 12 = 0$

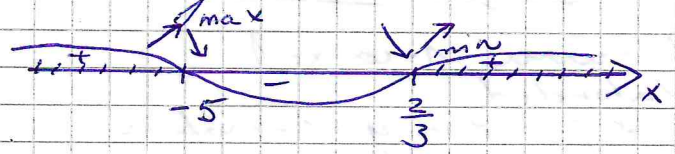
$x_{1,2} = \frac{-16 \pm \sqrt{256 + 4 \cdot 3 \cdot 12}}{2 \cdot 3} = \frac{-16 \pm \sqrt{400}}{6} = \frac{-16 \pm 20}{6}$

$x_1 = \frac{-16-20}{6} = -5$

$x_2 = \frac{-16+20}{6} = \frac{4}{6} = \frac{2}{3}$

$x_e \in \left\{-5; \frac{2}{3}\right\}$

* Liik → leian joonise abil




$x_{max} = -5$

$x_{min} = \frac{2}{3}$

c) * Kasvamisvahemik $X\uparrow$, tingimus $y' > 0$

leian jooniselt

$X\uparrow_1 = (-\infty; -5)$; $X\uparrow_2 = (\frac{2}{3}; \infty)$

* Kahanemisvahemik $X\downarrow$, tingimus $y' < 0$

$X\downarrow = (-5; \frac{2}{3})$

d) puutuja tõus kohal $x_0 = 1$

$k = f'(x_0)$

$k = f'(1) = 3 \cdot 1^2 + 16 \cdot 1 - 12 = 7$

(8) a) $f(x) = x^2 - 2x - 3$

* nullkohad $X_0$: tingimus $y = 0$

$x^2 - 2x - 3 = 0$

$x_{1;2} = \frac{2 \pm \sqrt{4 + 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 4}{2}$

$x_1 = \frac{2-4}{2} = -1$

$x_2 = \frac{2+4}{2} = 3$

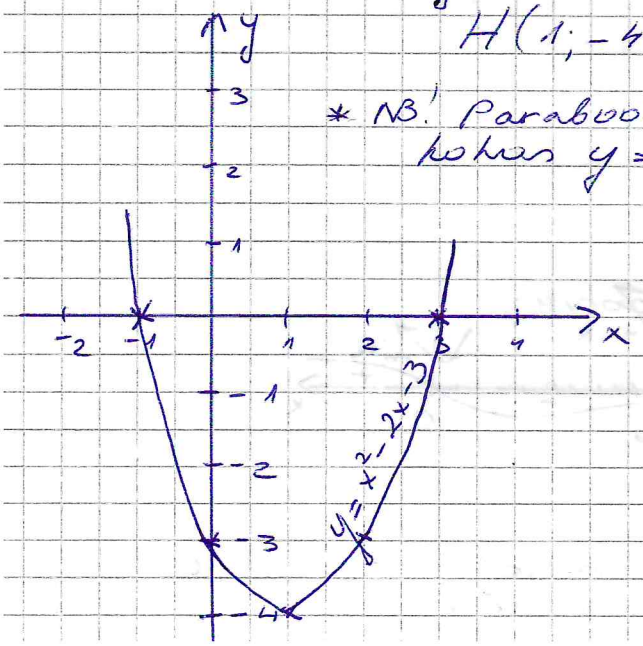
$X_0 = \{-1; 3\}$

* haripunkti koordinaadid $H(x_h; y_h)$

$x_h = \frac{x_1 + x_2}{2}$; $x_h = \frac{-1+3}{2} = 1$

$y_h = 1^2 - 2 \cdot 1 - 3 = -4$

$H(1; -4)$




* NB! Parabool avaneb ülal ja lõikab y-telge kohas $y = -3$

b) $g(x) = 1{,}5x^2 - 0{,}5x^3$

* ekstreemumkohad $x_e$: tingimus $y' = 0$

$y' = 3x - 1{,}5x^2$

$3x - 1{,}5x^2 = 0 \mid :(-1{,}5)$

$-2x + x^2 = 0$

$x^2 - 2x = 0$

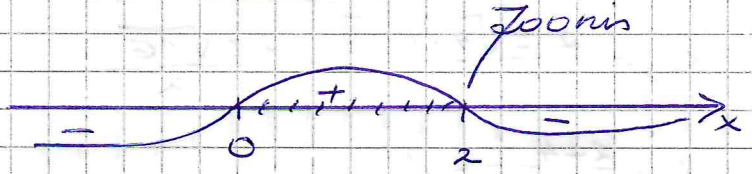
$x(x-2) = 0$

$x_1 = 0$ $\quad x - 2 = 0$, $x_2 = 2$

$x_e \in \{0;\ 2\}$

* Kasvamisvahemik $X\uparrow$, tingimus $y' > 0$

$-1{,}5x^2 + 3x > 0$




$X\uparrow = (0;\ 2)$

9) $f(x) = x^3 + 3x^2 + 2$

a) Kahanemisvahemik $X\downarrow$, tingimus $y' < 0$

* tuletis $y' = 3x^2 + 6x$

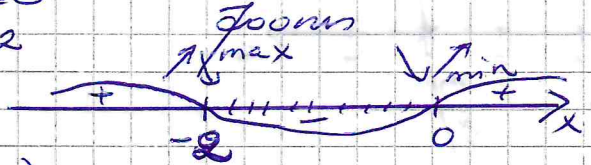
$3x^2 + 6x < 0$

$3x^2 + 6x = 0 \mid :3$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x_1 = 0$ $\quad x + 2 = 0$, $x_2 = -2$




$X\downarrow = (-2;\ 0)$

b) Miinimumpunkti koordinaadid $E_{min}(x; y)$

* Eelgi määran joonise abil

Kui $x_{min}=0$, siis $y_{min}=0^3+3\cdot0^2+2=2$

$E_{min}(0;2)$

c) Puutuja võrrand kohal $x_0=1$

* tõus $k=f'(x_0)$

$k=f'(1)=3\cdot1^2+6\cdot1=9$

* Puutepunkt $P(x_0,y_0)$

Kui $x_0=1$, siis $y_0=0^3+3\cdot0^2+2=2$

$P(1;2)$

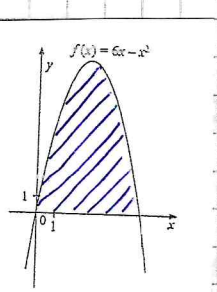
* puutuja võrrand $y=kx+b$

$b$? $9\cdot1+b=2$

$b=-7$

* $y=9x-7$

(10) $f(x)=6x-x^2$




a) * nullkohad $X_0$, tingimus: $y=0$

$6x-x^2=0 \mid\cdot(-1)$

$x^2-6x=0$

$x(x-6)=0$

$x_1=0$ $\quad x-6=0$

$x_2=6$

$X_0=\{0;6\}$

* haripunkti $H(x_k;y_k)$ koordinaadid

$x_k=\frac{x_1+x_2}{2}$; $x_k=\frac{0+6}{2}=3$

$y_k=6\cdot3-3^2=9$

$H(3;9)$

b) Viirutatud kujundi pindala

$$S=\int_0^6(6x-x^2)\,dx=\frac{6x^2}{2_1}-\frac{x^3}{3}\Big|_0^6=3x^2-\frac{x^3}{3}\Big|_0^6=$$

$$=3\cdot6^2-\frac{6^3}{3}-\left(3\cdot0^2-\frac{0^3}{3}\right)=108-72=36(ü^2)$$

(11) $f(x) = x^3 - 3x^2$

a) * Kasvamisvahemik $X\uparrow$; $y' > 0$

* tuletis $y' = 3x^2 - 6x$

$3x^2 - 6x > 0$

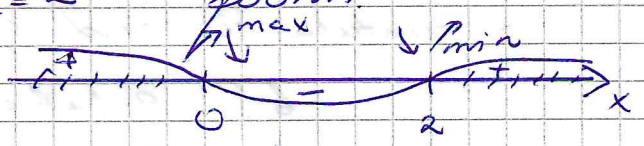
$3x^2 - 6x = 0 \mid :3$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x_1 = 0$ $\quad x - 2 = 0$

$x_2 = 2$




$X\uparrow_1 = (-\infty; 0)$; $X\uparrow_2 = (2; \infty)$

* Kahanemisvahemik $X\downarrow$; $y' < 0$

$X\downarrow = (0; 2)$

** Liik - määran joonise abil

$x_{min} = 2$, $y_{min} = 2^3 - 3 \cdot 2^2 = -4$

* Miinimumpunkt $E_{min}$ $(2; -4)$

b) Puutuja tõus kohal $x_0 = 3$

* tõus $k = f'(x_0)$

$k = f'(3) = 3 \cdot 3^2 - 6 \cdot 3 = 9$

(12) $f(x) = x^3 - 1{,}5x^2 - 6x$

a) *Kasvamisvahemik $X\uparrow$; tingimus $y' > 0$

* tuletis

$y' = 3x^2 - 3x - 6$

$3x^2 - 3x - 6 > 0$

$3x^2 - 3x - 6 = 0 \mid :3$

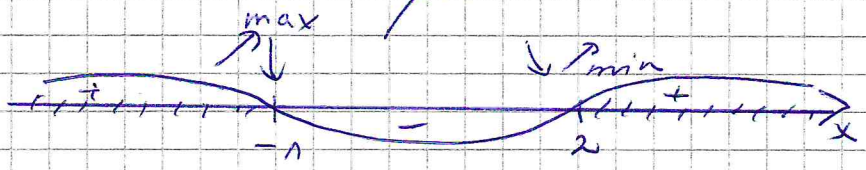
$x^2 - x - 2 = 0$

$x_{1,2} = \frac{1 \pm \sqrt{1 + 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2}$

$x_1 = \frac{1-3}{2} = -1$

$x_2 = \frac{1+3}{2} = 2$

joonis.




$X\uparrow_1 = (-\infty; -1)$ ; $X\uparrow_2 = (2; \infty)$

* Maksimumpunkti $E_{max}(x; y)$ koordinaadid leiga kiau jooniselt.

$x_{max} = -1$ $y_{max} = (-1)^3 - 1{,}5 \cdot (-1)^2 - 6 \cdot (-1) = 3{,}5$

$E_{max}(-1; 3{,}5)$

b) Puutuja võrrand kohal $x_0 = 1$

* tõus $k = y'(x_0)$

$k = y'(1) = 3 \cdot 1^2 - 3 \cdot 1 - 6 = -6$

* Puutepunkt $P(x_0; y_0)$

Kui $x_0 = 1$, siis $y_0 = 1^3 - 1{,}5 \cdot 1^2 - 6 \cdot 1 = -6{,}5$

$P(1; -6{,}5)$

* puutuja võrrand $y = kx + b$

$b$: $-6 \cdot 1 + b = -6{,}5$

$b = -0{,}5$

$y = -6x - 0{,}5$